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ON THE CLUSTER STRUCTURE OF A CIRCULATING FLUIDIZED BED

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A model of a bottom fluidized bed has been developed, in which the latter is represented as an ensemble of uniformly distributed spherical clusters. Using Ergun's formula, we obtained the dependence for calculating the diameter of a cluster. The results were compared with experimental data on the sizes of clusters in the transport zone of a circulating fluidized bed. In the framework of this model, a correlation to calculate the rate of filtration at which the bottom layer disappears has been obtained.

The structure of a circulating fluidized bed is characterized by the presence of clusters of particles, which are concentrated in the annular zone [1–4]. In [5], we obtained simple relations for the descending velocities of clusters and their vertical dimensions:

$$\frac{v_c}{u - u_t} = 0.1\ \mathrm{Fr}_t^{-0.7}\ , \tag{1}$$

$$\frac{L}{H} = 0.024\sqrt{1 - \varepsilon}\ . \tag{2}$$

Equations (1) and (2) describe the characteristics of clusters in the transport zone of a circulating fluidized bed $\left(\frac{x}{H} \gtrsim 0.1 - 0.2\right)$. As is known, a special zone is located near a gas-distributing grate, namely, the bottom fluidized bed. The characteristic feature of this bed is that at filtration velocities $u > u_t$ the concentration of particles in it is comparable with the concentration of particles in an ordinary fluidized bed, which cannot exist at all at such high velocities. Clearly, this is possible only because of the fact that the bottom fluidized bed is not an independent system but is a part of a more general straight-through one, such as the circulating fluidized bed is. The bottom fluidized bed may even be considered as a peculiar dynamic gas distributor, in which particles are accelerated and the thickness of which depends on the rate of filtration [6]:

$$\frac{H_0}{H} = 1.25\ \mathrm{Fr}_t^{-0.8}\ \overline{J}_s^{-1.1}\ . \tag{3}$$

Since high-power descending flows of particles enter into the bottom fluidized bed (near the walls of the standpipe) in the form of clusters, one may assume that the bottom fluidized bed is a collection of clusters of a certain effective diameter d_c related to the size of clusters in the lower part of the transport zone.

We were set the task of developing the cluster model of the bottom fluidized bed and deriving a computational relation for the cluster diameter similar to (2), and comparing them to prove the existence of a cause and effect relationship between the clusters in a bottom fluidized bed and in the transport zone of a circulating fluidized bed.

The bottom fluidized bed is considered as a flow subsystem, in which the particles arriving from both the peripheral region of the transport zone and the outer circulation loop are speeded up. To analyze the dynamics of the bottom fluidized bed, the following assumptions were made:

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(1) the cluster has a spherical shape and consists of the particles packed to a concentration of $1 - \varepsilon_{mf}$;

(2) the clusters are of the same size and are uniformly distributed in the space of the bottom fluidized bed with a concentration of $1 - \varepsilon_c$, which is related to the mean concentration of particles in the bottom fluidized bed:

$$1 - \varepsilon_c = \frac{1 - \varepsilon_{fb}}{1 - \varepsilon_{mf}}; \tag{4}$$

(3) the clusters move with acceleration vertically upward under the action of the friction force exerted by the gas; this friction force is described by the well-known Ergun formula [7];

(4) the gas velocity is constant over the standpipe section;

(5) the cluster velocity is changed linearly as it rises from 0 to $u - u_t$:

$$v_c = \frac{u - u_t}{H_0} x; \tag{5}$$

(6) gas filtration through the cluster is ignored.

We will write the equation of Newton's second law for an individual cluster:

$$\frac{\pi d_c^3}{6} \rho_s (1 - \varepsilon_{mf}) \frac{dv_c}{dt} = -\frac{\pi d_c^3}{6} \rho_s (1 - \varepsilon_{mf}) g + F_c. \tag{6}$$

The force acting on the cluster from the side of the gas flow is calculated by the equation

$$F_c = \frac{\Delta p}{\Delta x} \Big/ n, \tag{7}$$

where n is the number of clusters per unit volume of the bottom fluidized bed:

$$n = \frac{6(1 - \varepsilon_c)}{\pi d_c^3}. \tag{8}$$

The value of $\frac{\Delta p}{\Delta x}$, subject to assumption (3), is calculated by the Ergun formula [6]:

$$\frac{\Delta p}{\Delta x} = 150 \cdot \frac{(1 - \varepsilon_c)^2}{\varepsilon_c^3} \frac{\mu_f u_r}{d_c^2} + 1.75 \cdot \frac{(1 - \varepsilon_c)}{\varepsilon_c^3} \frac{\rho_f u_r^2}{d_c}. \tag{9}$$

With account for Eqs. (8) and (9), we will obtain the following equation for F_c:

$$F_c = \frac{\pi \mu_f^2}{\rho_f} \left(25 \cdot \frac{1 - \varepsilon_c}{\varepsilon_c^3} \mathrm{Re}_r + 0.29 \cdot \frac{1}{\varepsilon_c^3} \mathrm{Re}_r^2 \right). \tag{10}$$

Then, the equation of motion of the cluster will have the form

$$\frac{dv_c}{dt} = -g + A_0 \left(150 \cdot \frac{1 - \varepsilon_c}{\varepsilon_c^3} \mathrm{Re}_r + 1.75 \cdot \frac{1}{\varepsilon_c^3} \mathrm{Re}_r^2 \right). \tag{11}$$

In order to estimate the size of the cluster, we will average Eq. (11) over the longitudinal coordinate:

$$\left\langle \frac{dv_c}{dt} \right\rangle = \frac{1}{H_0} \int_0^{H_0} \frac{dv_c}{dt}\, dx \,, \tag{12}$$

$$\langle \mathrm{Re_r} \rangle = \frac{1}{H_0} \int_0^{H_0} \mathrm{Re_r}\, dx \,, \tag{13}$$

$$\langle \mathrm{Re_r^2} \rangle = \frac{1}{H_0} \int_0^{H_0} \mathrm{Re_r^2}\, dx \,. \tag{14}$$

The value of $\left\langle \frac{dv_c}{dt} \right\rangle$ will be found independently based on the following arguments. We will consider a change in the concentration of labeled particles in the bottom fluidized bed in the absence of external circulation of these particles:

$$\rho_{fb} H_0 \frac{dc_{fb}}{dt} + (A\rho_1 u_1 c_1 - B\rho_2 u_2 c_2)\big|_{x=H_0} = 0 \,. \tag{15}$$

Equation (15), written thus, presupposes a good mixing of particles in the bottom fluidized bed. The dimensionless concentration c_1 is causally related to the quantity c_{fb}; therefore it can justifiably be assumed that $c_{fb} = c_1$. Thereafter, Eq. (15) can be simplified substantially if we take into account the fact that in the region $x = H_0$ there is a substantial horizontal transfer of particles. This allows the assumption that $c_1 \approx c_2 = c$, and Eq. (15) can be presented in the form

$$\rho_{fb} H_0 \frac{dc}{dt} + J_s c = 0 \,, \tag{16}$$

taking into account the fact that $J_s = A\rho_1 u_1 - B\rho_2 u_2$. The solution of Eq. (16) is as follows:

$$c = \exp\left(- t / \langle t \rangle\right) \,, \tag{17}$$

where $\langle t \rangle = \frac{\rho_s (1 - \varepsilon_{fb}) H_0}{J_s}$ is the mean time of residence of fine particles in the bottom fluidized bed. Since this time characterizes the dynamics of the motion of clusters in the bottom fluidized bed, for $\left\langle \frac{dv_c}{dt} \right\rangle$ we obtain

$$\left\langle \frac{dv_c}{dt} \right\rangle = \frac{u - u_t}{\langle t \rangle} = \frac{(u - u_t) J_s}{\rho_s (1 - \varepsilon_{fb}) H_0} \,. \tag{18}$$

To calculate the mean relative filtration rate $\langle u_r \rangle$ entering into the expression $\langle \mathrm{Re_r} \rangle$, we will preliminarily write the following equation:

$$\frac{u}{\varepsilon_c} - v_c = v_r \,; \tag{19}$$

which defines the relative velocity of the gas in the space between the clusters. With account for Eqs. (5) and (19), for $\langle u_r \rangle = \langle \varepsilon_c v_r \rangle$ we obtain

$$\langle u_r \rangle = \langle u - \varepsilon_c v_c \rangle = u - \varepsilon_c \langle v_c \rangle = u - \varepsilon_c \frac{u - u_t}{2} . \tag{20}$$

For $\langle u_r^2 \rangle$ we have

$$\langle u_r^2 \rangle = \frac{1}{H_0} \int_0^{H_0} \left(u - \varepsilon_c \frac{u - u_t}{H_0} x \right)^2 dx = \langle u_r \rangle^2 + \frac{1}{12} \varepsilon_c^2 (u - u_t)^2 . \tag{21}$$

Comparison of (20) and (21) under the conditions typical of the circulating fluidized bed lead to the approximate equality

$$\langle u_r^2 \rangle = \langle u_r \rangle^2 . \tag{22}$$

Subject to Eq. (22), Eq. (11) will have the following form after averaging:

$$\frac{(u - u_t) J_s}{\rho_s (1 - \varepsilon_{fb}) H_0} = - g + A_0 \left(150 \cdot \frac{1 - \varepsilon_c}{\varepsilon_c^3} \langle \mathrm{Re}_r \rangle + 1.75 \cdot \frac{1}{\varepsilon_c^3} \langle \mathrm{Re}_r \rangle^2 \right). \tag{23}$$

As estimates show, for the conditions of operation of the circulating fluidized bed the term that is linear in $\langle \mathrm{Re}_r \rangle$ in Eq. (23) can be neglected. Then for $\langle \mathrm{Re}_r \rangle = \langle u_r \rangle / d_c / \nu_f$ Eq. (23) yields

$$\langle \mathrm{Re}_r \rangle = \sqrt{\frac{\varepsilon_c^3}{1.75 B_0 (1 - \varepsilon_c)} (g \rho_s (1 - \varepsilon_{fb}) + \frac{(u - u_t) J_s}{H_0}} . \tag{24}$$

Considering that $B_0 \sim d_c^{-3}$, we obtain the needed dependence to determine the cluster diameter:

$$\frac{d_c}{H} = 1.75\, \mathrm{Fr}_t^* \frac{\rho_f}{\rho_s} \frac{1 - \varepsilon_c}{\varepsilon_c^3 (1 - \varepsilon_{fb}) \left(1 + \frac{(u - u_t) J_s}{H_0 g \rho_s (1 - \varepsilon_{fb})} \right)} . \tag{25}$$

As estimates show, $(u - u_t) J_s / H_0 g \rho_s (1 - \varepsilon_{fb}) << 1$.*) This indicates that the influence of inertia forces can be neglected.

The quantity $1 - \varepsilon_{fb}$ is rather stable [6]:

$$1 - \varepsilon_{fb} = 1 - 0.33\, \mathrm{Fr}_t^{-0.045} \approx 0.35 . \tag{26}$$

Here we note that the practical independence of the bottom fluidized-bed porosity of the filtration rate seems rather surprising. But precisely this phenomenon allows us to consider the acceleration of clusters in the bottom fluidized bed and give a description in the framework of the given model. In the classical fluidized bed this seems to be impossible, since the particles in that layer move apart from each other on increase in the gas velocity (the bed is expanded), the new friction force again counterbalances the weight of the particle, and on the average its acceleration is absent.

Considering $1 - \varepsilon_{mf} \approx 0.6$ and Eqs. (4) and (26), to determine the cluster diameter on the basis of (25) we have

$$\frac{d_c}{H} \approx 40.4\, \mathrm{Fr}_t^* \frac{\rho_f}{\rho_s} . \tag{27}$$

To verify Eq. (26), we will compare the calculations performed by Eq. (26) and by (2). Preliminarily, we will write Eq. (2) with account for the empirical correlation from [8]:

*) This relation is violated in the region of $u \approx u_*$, when $H_0 \to 0$ (see below).

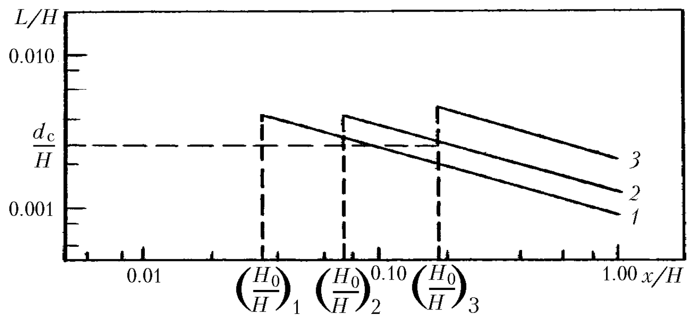

Fig. 1. Dimensions of clusters in the transport zone of the circulating and bottom fluidized beds: 1) J_s = 10; 2) 20; 3) 50 kg/(m^2·sec).

$$1 - \varepsilon = \bar{J}_s \left(\frac{x}{H}\right)^{-0.82} , \tag{28}$$

$$\frac{L}{H} = 0.024 \, \bar{J}_s^{0.5} \left(\frac{x}{H}\right)^{-0.41} . \tag{29}$$

We will perform calculations for the following specific conditions: d = 0.32 mm, u_t = 2.8 m/sec, u = 4.5 m/sec, H = 13.5 m, ρ_s = 2600 kg/m^3, and ρ_f = 1.3 kg/m^3. The results of calculation for different J_s (the region of verification of Eq. (29)) are shown in Fig. 1. It also gives the value of d_c = 0.035 m $\left(\frac{d_c}{H} = 0.026\right)$ obtained from Eq. (27). We note that the empirical equation (29) is valid only for the transport zone of the circuiting fluidized bed $\frac{x}{H} \gtrsim 0.1 - 0.2$. As is seen, the values of L in the region of $\frac{x}{H} \approx 0.1$–0.2 agree satisfactorily with the value of d_c, which evidently points to the validity of the proposed approach toward estimation of the size of the cluster in the bottom fluidized bed. The resulting equation (27) can be used for independent estimation of the dimensions of the clusters in the lower part of the circulating fluidized bed, where Eqs. (2) and (29) are inapplicable.

As is seen from Eq. (27), the value of d_c increases with the rate of filtration. In connection with this, the velocity at the onset of fluidization of the cluster is also increased. We will estimate the dependence of the number of fluidizations of the cluster $N_c = u/u_{mf}^c$ on the rate of filtration. The velocity u_{mf}^c is calculated from the well-known equation of Todes [9], and the cluster diameter is determined from Eq. (27); the density of the cluster was assumed equal to $\rho_s(1 - \varepsilon_{mf})$. The velocity u_*, at which the bottom fluidized bed disappears, was estimated from the empirical formula [10]

$$\frac{u_* - u_t}{u_t} = 5.0 \left(\frac{J_s}{\rho_s u_t}\right)^{0.4} \left(\frac{u_t^2}{gH}\right)^{-0.3} . \tag{30}$$

The calculations performed gave: N_c = 1.08, 1.1, 1.16, and 1.2 for u = 2.8, 3.0, 4.5, and 6 m/sec, respectively (u_t = 2.8 m/sec, u_* = 7.2 m/sec).

As is seen, in the entire range of filtration rates in which the bottom fluidized bed exists, the numbers of fluidizations of the cluster differ little from unity, although the acting rates of filtrations may be several times higher than the velocity of traveling of a single particle.

Thus, in the proposed model the phenomenon of the bottom fluidized bed finds a simple explanation: not individual particles are fluidized in the bottom fluidized bed, but rather their large agglomerates (clusters), the rate of the onset of the fluidization of which is not much less than the acting rate of gas filtration.

Based on the relation derived to calculate the size of the cluster (27), one may obtain a practically important characteristic of the circulating fluidized bed, namely, the velocity at which the bottom fluidized bed disappears (u_*).

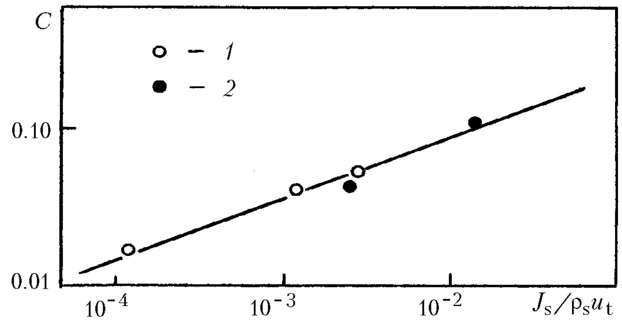

Fig. 2. Rate of filtration at which the bottom fluidized bed disappears: 1) [11], $\rho_f = 0.31$ kg/m^3; 2) [12], $\rho_f = 1.2$ kg/m^3. Solid line, calculation by Eq. (34). $C = (u_* - u_t)/(u_t(\rho_s/\rho_f)^{0.22}(u_t^2/gH)^{-0.38})$.

We will make a natural assumption: at $u = u_*$ the value of d_c becomes commensurable with the height of the bottom fluidized bed, i.e., $d_c \approx H_0$. Using Eqs. (3), (25), and (27), we will write this condition in the form

$$40.4\,\mathrm{Fr}_t^* \frac{\rho_f}{\rho_s} \frac{1}{\left(1 + \dfrac{(u - u_t)\,J_s}{H_0 g \rho_s\,(1 - \varepsilon_{fb})}\right)} = 1.25\,\mathrm{Fr}_t^{-0.8}\,J_s^{-1.1}\,. \tag{31}$$

Equation (31) will be transformed to

$$\frac{\left(u_* - \varepsilon_c \dfrac{u_* - u_t}{2}\right)^2 (u_* - u_t)^{2.7}}{1 + \dfrac{(u_* - u_t)\,J_s}{H_0 g \rho_s\,(1 - \varepsilon_{fb})}} = 0.031\,(gH)^{1.8}\,J_s^{1.1} \rho_s^{-0.1} \rho_f^{-1}\,. \tag{32}$$

Assuming approximately that $(u_* - \varepsilon_c(u_* - u_t)/2)^2/(1 + (u_* - u_t)J_s/H_0 g\rho_s(1 - \varepsilon_{fb})) \sim (u_* - u_t)^2$, from Eq. (32) we obtain the dependence of the quantity $(u_* - u_t)/u_t$ on the determining parameters:

$$\frac{u_* - u_t}{u_t} \simeq 0.47 \left(\frac{J_s}{\rho_s u_t}\right)^{0.24} \left(\frac{u_t^2}{gH}\right)^{-0.38} \left(\frac{\rho_s}{\rho_f}\right)^{0.22}\,. \tag{33}$$

Processing of the experimental data in terms of u_* [11, 12] on the basis of Eq. (33) made it possible to perform a certain verification (the exponent of ρ_s/ρ_f was *a priori* taken equal to 0.22):

$$\frac{u_* - u_t}{u_t} = 0.47 \left(\frac{J_s}{\rho_s u_t}\right)^{0.38} \left(\frac{u_t^2}{gH}\right)^{-0.38} \left(\frac{\rho_s}{\rho_f}\right)^{0.22} = 0.47 \left(\frac{J_s g H}{\rho_s u_t^3}\right)^{0.38} \left(\frac{\rho_s}{\rho_f}\right)^{0.22}\,. \tag{34}$$

Figure 2 presents a comparison of the experimental data of [11, 12] with those calculated by Eq. (34). Comparison of Eq. (34) with Eq. (30) obtained earlier in [10] shows that Eq. (34), which takes into account the influence of the simplex ρ_s/ρ_f, may be considered as a generalization of Eq. (30). It allows one to predict the values of u_* at elevated temperatures and pressures. We also note that formally, from the viewpoint of similarity theory, the value of the coefficient 0.47 in Eq. (34) is better than 5.0 in Eq. (30). The number $J_s gH/\rho_s u_t^3$ that determines the value of $(u_* - u_t)/u_t$ in Eq. (34) is the ratio of the gas-flow powers spent to raise particles and to keep them in the gravity field.

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NOTATION

A, fraction of the horizontal section occupied by ascending particles in the standpipe (core of the bed); $A_0 = \mu_f^2/\rho_s(1-\varepsilon_{mf})d_c^3\rho_f$; B, fraction of the horizontal section occupied by descending particles in the standpipe (annular zone); $B_0 = \mu_f^2/d_c^3\rho_f$; c_1, c_2, and c_{fb}, dimensionless concentrations of labeled particles in the bed core, in the annular zone, and in the bottom fluidized bed, respectively; d_c, diameter of the cluster in the bottom fluidized bed, m; d, diameter of particles, m; F_c, resistance force, kg·m/sec^2; $\mathrm{Fr_t} = (u-u_t)^2/gH$, $\mathrm{Fr_t^*} = \left(u - \varepsilon_c \frac{u-u_t}{2}\right)^2 \Big/ gH$, the Froude number; g, free-fall acceleration, m/sec^2; H, height of the standpipe, m; H_0, height of the bottom bed, m; J_s, outer circulating flow, kg/(m^2·sec); $\bar{J}_s = J_s/\rho_s(u-u_t)$; L, vertical dimension of the cluster in the transport zone, m; $\frac{\Delta p}{\Delta x}$, pressure difference at the height Δx, kg/(m·sec^2); $\mathrm{Re_r} = u_r d_c/\nu_f$, Reynolds number; t, time, sec; u, rate of gas filtration, m/sec; u_{mf}^c, rate of the onset of fluidization of the cluster, m/sec; u_r, relative rate of filtration, m/sec; u_t, rate of traveling of a single particle, m/sec; u_1 and u_2, velocities of particles in the bed core and annular zone, respectively, m/sec; u_*, velocity at which the bottom layer disappears, m/sec; v_c, velocity of the cluster, m/sec; x, longitudinal coordinate, m; ε, porosity in the transport zone; ε_{fb}, mean porosity of the bottom bed; ε_c, porosity of the bed consisting of clusters of diameter d_c; ε_{mf}, porosity of the cluster; μ_f, dynamic porosity of the gas, kg/(m·sec); ν_f, kinematic porosity of the gas, m^2/sec; ρ_f and ρ_s, density of the gas and particles, respectively, kg/m^3; ρ_1 and ρ_2, concentration of particles in the core of the bed and annular zone, respectively, kg/m^3; ρ_{fb}, density of the bottom bed, kg/m^3. Subscripts and superscripts: 1, core of the bed; 2, annular zone; c, cluster; f, gas; fb, bottom bed; mf, onset of fluidization; r, relative; s, particle; t, conditions to the travel of a solitary particle.

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